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On the Mass-Loss Hypothesis of Virial Mass Discrepancy in Groups and Clusters of Galaxies

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Abstract

The virial mass discrepancy in groups and clusters of galaxies is reexamined from the viewpoint of the mass-loss hypothesis. The relation between the virial mass discrepancy and the density of the system is derived, and the general features of observations are well explained. This relation implies that the system of galaxies is unbound and expands freely.

Key words: Clusters of galaxies; Mass-loss hypothesis; Virial theorem.

1. *Introduction*

Application of the virial theorem to groups and clusters of galaxies showed that the virial mass, M_V, is at least an order of magnitude greater than the probable sum of the masses of individual galaxies (Burbidge and Sargent 1971; de Vaucouleurs 1975; Gott and Turner 1977). This problem is called the virial mass discrepancy of the missing mass. The most commonly proposed explanations are: (1) that the groups and clusters are unbound by nature (Ambartsumian 1958), (2) that the clusters contain much unseen matter such as black dwarfs (Napier and Guthrie 1975) and black holes (Schmidt and Oleak 1971), and (3) that the clusters are disintegrating through mass loss (Noerdlinger 1970; Field and Saslaw 1971; Aarseth and Saslaw 1972).

On the mass-loss hypothesis, Aarseth and Saslaw (1972) have studied, and a range of mass discrepancies consistent with observations (Rood et al. 1970) has been reproduced from relatively moderate assumptions. On the other hand, Ozernoi (1975) has refuted this hypothesis on account of the disagreement with observations by Karachentsev (1970). However, Hartwick (1978) recently has presented evidence that the virial mass discrepancy depends on the density of the system. That is, among the high-density groups ($\bar{\rho}>10^{-27}\,\mathrm{g\,cm^{-3}}$) the virial mass is independent of the density of the group, and among the low-density groups ($\bar{\rho}<10^{-28}\,\mathrm{g\,cm^{-3}}$) the ratio of the virial mass to the observed mass increases with decreasing density. In the latter groups, the observational velocity is proportional to the radius of gyration of the group.

Thus, it is important to reexamine the mass-loss hypothesis referring to the evidence presented by Hartwick (1978).

The mechanisms of mass loss from a system of galaxies will occur through some dynamical phenomena, especially, from the nuclei of massive galaxies. Two specific physical mechanisms have been proposed. One is that a blast wave emanating from a quasar in a cluster of galaxies could eject the intergalactic gas from the cluster (Noerdlinger 1970). Similar phenomena could also be expected in the early phase of galaxies if the star formation rate and the subsequent supernova-explosion rate were much higher than those at the present time (Ikeuchi 1977). These mechanisms in which a large amount of matter is directly ejected seem promising (Yahil and Ostriker 1973; Cowie and Perrenod 1978; Hirayama et al. 1978). The second possibility is that matter is lost in the form of gravitational radiation (Field et al. 1969) or high-energy neutrinos from massive or primordial black holes (Sciama 1976). Although these mechanisms are not confirmed in the present time, it cannot be denied that the energy of the universe may be in the form of gravitational radiation or massless particles.

In any case, the time scales and duration times of mass loss would be expected to vary among groups of galaxies, and the observed range of mass discrepancies would show these variations.

Here, we investigate the expansion law of the group through mass loss (section 2), and the evolution tracks on the diagram of the virial mass discrepancy versus the density of the group are calculated (section 3). For various parameters of the mass-loss time and the initial crossing time, the correlations between the virial mass discrepancy and the density of the groups presented by Hartwick (1978) are reproduced. In section 4, some discussions are given.

2. *Expansion Law of a Cluster*

The virial mass of a cluster is determined in the form

$$M_{\rm V}=V^2R_\Omega/G\ , \tag{1}$$

in which G is the gravitational constant, V is the velocity dispersion of galaxies, and R_Ω is the mass-weighted harmonic mean radius. Later we simplify the cluster model such that equal-mass galaxies are distributed uniformly and spherically within the radius R_0. In this case, the relations among R_Ω, R_0, and the radius of gyration of the group, $R_{\rm I}$, are

$$R_\Omega=(5/3)R_0 \qquad \text{and} \qquad R_{\rm I}=(3/5)^{1/2}R_0\ . \tag{2}$$

These relations are assumed to hold for all times. The density of the system is

$$\bar{\rho}=3M/4\pi R_0^3=3M/4\pi R_{\rm I}^2R_\Omega\ , \tag{3}$$

which is the same as defined in Hartwick's (1978) paper.

On the mass-loss rate, we assume as

$$\dot{M}=-M/T_l\ , \tag{4}$$

where T_l is the characteristic time for mass loss. As shown by Aarseth and Saslaw (1972), different laws of mass-loss rate would not give much difference in the virial mass calculation so long as the characteristic time of mass loss is of the same order.

The dynamical evolution of a system of galaxies depends upon the relation between T_l and the crossing time $T_{\rm c}=R_{\rm I}/V=(3/5)^{1/2}R_0/V=(3/5)^{3/2}R_\Omega/V$ (Jackson 1970). If $T_{\rm c}<T_l$, the system will expand adiabatically and the virial theorem will hold

when $T_c < t_0$, t_0 being the age of the system. If $T_c > T_l$ it will expand freely and the virial mass discrepancy will arise. The binding energy of the system, E_B, is written as

$$E_B = \frac{1}{2} M V^2 - \frac{GM^2}{R_\Omega} = \frac{GM^2}{2R_\Omega}\left(\frac{M_V}{M} - 2\right) . \tag{5}$$

Thus, the system of $M_V/M > 2$ becomes unbound and expands without limit.

In the case of adiabatic expansion, $V \propto R_\Omega^{-1}$, the virial theorem implies that $M = M_V$ decreases as R_Ω^{-1}. Thus, the relation holds as

$$\frac{V}{V_i} = \frac{R_{\Omega,i}}{R_\Omega} = \frac{M}{M_i} = \exp\left[\frac{-(t-t_i)}{T_l}\right] , \tag{6}$$

where the subscript i means the values at the initial time t_i. From equation (6) the crossing time increases as

$$T_c = T_{c,i}(R_\Omega / R_{\Omega,i})^2 = T_{c,i} \exp\left[2(t-t_i)/T_l\right] . \tag{7}$$

On the other hand, at the free expansion phase the velocity dispersion remains constant and the radius becomes

$$R_0 = R_{0,i} + V(t-t_i) . \tag{8}$$

If the virial theorem holds at the initial time, the virial mass increases as

$$M_V = M_i R_\Omega / R_{\Omega,i} = M_i [1 + (t-t_i)/T_{c,i}] . \tag{9}$$

Since the mass of the system varies as $M = M_i \exp[-(t-t_i)/T_l]$, the ratio M_V/M increases as

$$\frac{M_V}{M} = \left(1 + \frac{t-t_i}{T_{c,i}}\right) \exp\left(\frac{t-t_i}{T_l}\right) . \tag{10}$$

Thus, it can be considered that the virial mass discrepancy increases at the free expansion phase through mass loss. (Here, the mass of the system M is assumed to be the same as the observed mass.)

3. *Dynamical Evolution of a Cluster*

3.1. *Case A: The Case of Continuous Mass Loss*

We assume that the virial theorem holds at the initial time ($t_i = 0$). This is guaranteed when

$$T_{c,i} \equiv (3/5)^{3/2} R_0 / V_i < t_0 . \tag{11}$$

When $T_{c,i} < T_l$, the system begins to expand adiabatically and the crossing time increases as in equation (7). At the epoch of

$$t = t_T = \frac{1}{2} T_l \ln\left(\frac{T_l}{T_{c,i}}\right) , \tag{12}$$

T_c coincides with T_l and after that, T_l becomes smaller than T_c. We call this epoch the transition time and the physical quantities at this time are denoted by the subscript T. The virial mass discrepancy arises if $t_T < t_0$, i.e.,

$$\frac{T_l}{T_{c,i}} \leq \exp\left(\frac{2t_0}{T_l}\right) . \tag{13}$$

The physical quantities at this stage are written as

$$\frac{V_T}{V_i} = \frac{M_T}{M_i} = \frac{R_{\Omega,i}}{R_{\Omega,T}} = \left(\frac{T_{c,i}}{T_l}\right)^{1/2} . \tag{14}$$

After this stage, the system expands freely and the virial mass discrepancy increases as

$$\frac{M_V}{M} = \left(1 + \frac{t - t_T}{T_l}\right) \exp\left(\frac{t - t_T}{T_l}\right) . \tag{15}$$

In figure 1, the time variations of M_V/M are illustrated for various values of $\alpha \equiv T_l/T_{c,i}$.

In comparison with the results by Aarseth and Saslaw (1972), the virial mass discrepancy increases more rapidly for the same α. The reason is that the approximation of free expansion at $0.2T_c \lesssim T_l \lesssim T_c$ is not so good and the system goes through a quasi-adiabatic stage. Thus, our results show the upper limits of virial mass.

At the present time $t = t_0$, the crossing time becomes

$$T_{c,0} = t_0 + T_l - t_T . \tag{16}$$

From the condition of $0 \leq t_T \leq t_0$, the upper and lower limits of $T_{c,0}$ and M_V/M are easily obtained:

$$T_l \leq t_{c,0} \leq t_0 + T_l \tag{17}$$

and

$$1 \leq \frac{M_V}{M} \leq \left(1 + \frac{t_0}{T_l}\right) \exp\left(\frac{t_0}{T_l}\right) . \tag{18}$$

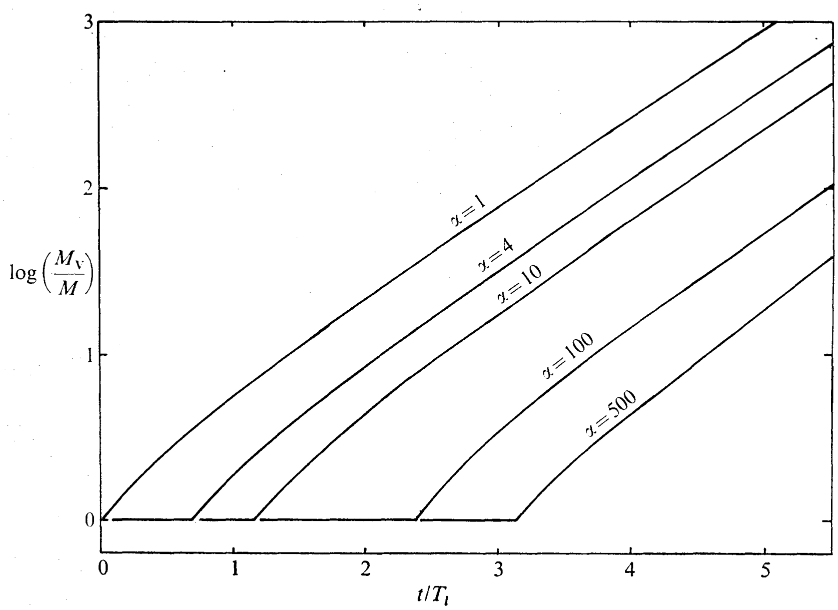

Fig. 1. The time variation of the virial mass discrepancy for various values of $\alpha = T_l/T_{e,i}$. The time is normalized by T_l.

The present density of the system becomes

$$\frac{\bar{\rho}_0}{\bar{\rho}_\mathrm{i}}=\frac{M}{M_i}\left(\frac{R_{0,\mathrm{i}}}{R_0}\right)^3=\left(1+\frac{t_0-t_\mathrm{T}}{T_l}\right)^{-3}\exp\left[\frac{-(t_0+3t_\mathrm{T})}{T_l}\right]. \tag{19}$$

From the definitions of $\bar{\rho}$ and T_c, equation (19) is rewritten as

$$\bar{\rho}_0=\frac{3}{4\pi G}\left(\frac{1}{T_{c,\mathrm{i}}}\right)^2\left(\frac{T_l}{T_{c,0}}\right)^3\exp\left[\frac{-(t_0+3t_\mathrm{T})}{T_l}\right]. \tag{20}$$

From equation (15) at $t=t_0$ and equation (20), we can easily obtain the relation

$$\frac{M_\mathrm{V}}{M}=\frac{3}{4\pi G}\frac{H_\mathrm{eff}{}^2}{\bar{\rho}_0}, \tag{21}$$

where

$$H_\mathrm{eff}=T_{c,0}^{-1}\cong 1.5H_0[1+(T_l-t_\mathrm{T})/t_0]^{-1}. \tag{22}$$

Here, H_0 is the present Hubble expansion rate and $t_0\simeq 1.5H_0{}^{-1}$ is assumed. The relation $V=H_\mathrm{eff}R_\mathrm{I}=T_{c,0}^{-1}R_\mathrm{I}$ is naturally obtained. These relations are presented by Hartwick (1978), and he has determined the range of H_eff to be from 87.1 to 181 km s^{-1} Mpc^{-1} in the low density groups. This implies, from equation (22),

$$5.5\times10^9\ \mathrm{yr}\leq T_{c,0}\leq 1.2\times10^{10}\ \mathrm{yr}. \tag{23}$$

Corresponding restrictions on $T_{c,0}$ are already obtained by equation (17) in our model. If we set $T_l=5.5\times10^9$ yr as the left-hand side inequality in (23), a larger value of $T_{c,0}$ (a smaller value of H_eff) is allowed in our model as the right-hand inequality in expression (17). This means that the virial mass discrepancy of a cluster, whose present crossing time is longer than the age of the universe, is also explained naturally in our model.

On the other hand, in the high-density groups the condition (17) is not satisfied because the initial crossing time is so short: $t_\mathrm{T}>t_0$. Thus, the value of M_V/M does not deviate from unity. These general features derived from our model naturally explain the evidence presented by Hartwick (1978).

In reality, however, we cannot determine T_l definitely, but its allowed range will be estimated as follows: The upper limit of T_l is obtained in equation (13) as $T_{c,\mathrm{i}}=t_0$, since the virial mass discrepancy due to mass loss demands free expansion within the age of the group. On the other hand, the lower limit of T_l is determined from the consideration of the cosmic mass density. The mass density of all the observed galaxies is as low as $\rho_c/100$, ρ_c being the critical density of the universe. The mass ejected from clusters will be at most this critical value, i.e., $(M_\mathrm{i}-M)\leq 0.99M_\mathrm{i}$. Thus, we obtain $T_l\geq t_0/4.6$.

In figure 2, the M_V/M-$\bar{\rho}_0$ diagram is illustrated for $t_0=1.3\times10^{10}$ yr and various values of T_l and α. In this figure, the range of H_eff is 38.5 ($T_l=1.3\times10^{10}$ yr) to 345 ($T_l=2.9\times10^9$ yr) km s^{-1} Mpc^{-1}.

The dispersions of M_V/M in low-density groups in Hartwick's (1978) figure would arise from different values of T_l and α for individual clusters.

3.2. *Cases B and C: The Cases of Finite Interval of Mass-Loss Phase*

Generally, the mass-loss mechanism will work effectively when a system of galaxies is denser or the individual galaxies are active. Then, we examine such

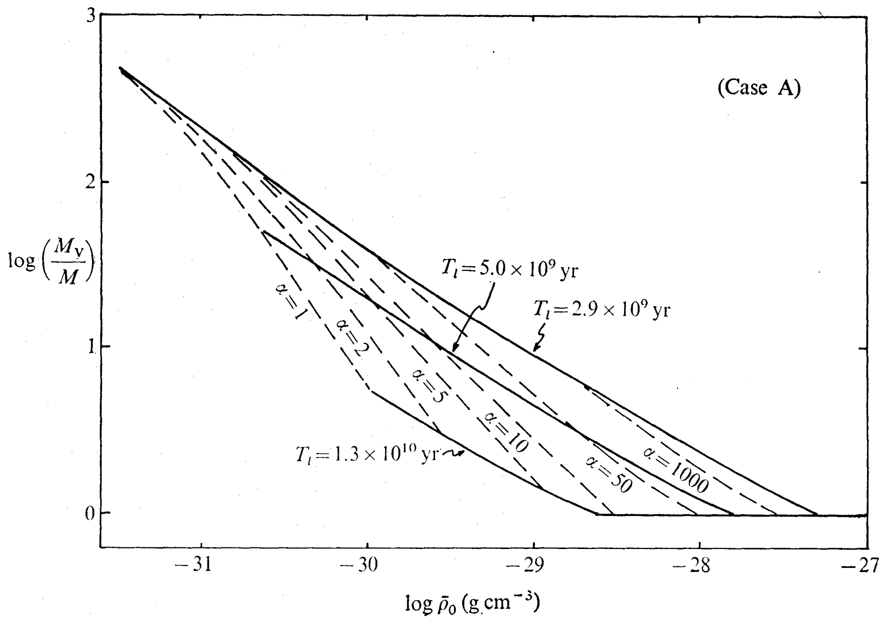

Fig. 2. The relation between the virial mass discrepancy and the density of the system in Case A. The solid lines represent constant T_l and the dashed lines show the contours of constant α.

a model that the mass loss occurs only for a finite time interval on a short time scale.

As a simplified model, we consider that the mass loss occurs in the era of $t_1 \leq t \leq t_2$. At the time of $0 \leq t \leq t_1$, the system of galaxies is bound and the virial theorem holds. Thus, the crossing time at $t=t_1$ must be

$$T_{c,1} \leq t_1 \ . \tag{24}$$

At the time of $t_2<t<t_0$, the mass loss stops but the system continues to expand freely if $E_B(t=t_2)>0$, i.e., $(M_V/M)_{t=t_2}>2$.

The analysis of this model for $t_1 \leq t \leq t_2$ is essentially the same as for Case A, so long as the initial condition is set at the time t_1. Then, M_V/M at $t=t_2$ is written as

$$\left(\frac{M_V}{M}\right)_{t=t_2} \begin{cases} =\dfrac{T_{c,2}}{T_l} \exp\left(\dfrac{t_2-t_T{}^1}{T_l}\right) & \text{for} \quad T_{c,1}<T_l \ , & (25) \\[2ex] =\dfrac{T_{c,2}}{T_{c,1}} \exp\left(\dfrac{t_2-t_1}{T_l}\right) & \text{for} \quad T_{c,1}>T_l \ , & (26) \end{cases}$$

where

$$T_{c,2} \begin{cases} =t_2+T_l-t_T{}^1 & T_{c,1}<T_l \ , & (27) \\ =t_2+T_{c,i}-t_1 & T_{c,1}>T_l \ , & (28) \end{cases}$$

and

$$t_T{}^1=t_1+\frac{T_l}{2} \ln\left(T_l/T_{c,1}\right) \ . \tag{29}$$

The present values of M_V/M and $T_{c,0}$ for the system of $(M_V/M)_{t=t_2}>2$ are given as

$$\frac{M_V}{M}\begin{cases} =\dfrac{T_{c,0}}{T_l}\exp\left(\dfrac{t_2-t_T{}^1}{T_l}\right) & \text{for} \quad T_{c,1}<T_l\,, \quad (30) \\[2ex] =\dfrac{T_{c,0}}{T_{c,1}}\exp\left(\dfrac{t_2-t_1}{T_l}\right) & \text{for} \quad T_{c,1}>T_l\,, \quad (31) \end{cases}$$

and

$$T_{c,0}\begin{cases} =t_0+T_l-t_T{}^1 & \text{for} \quad T_{c,1}<T_l\,, \quad (32) \\ =t_0+T_{c,1}-t_1 & \text{for} \quad T_{c,1}>T_l\,, \quad (33) \end{cases}$$

respectively. On the other hand, when $(M_V/M)_{t=t_2}<2$, the system is a bound one and it will evolve to the virial stage ($M_V/M=1$) with the time scale $T_{c,2}$. As the parameters t_1 and t_2 are not known, we study two extreme cases of $T_{c,1}\leq t_1=T_l$ and $T_l\leq T_{c,1}\leq t_1=10T_l$.

As the first simple model (Case B), we consider the case of $t_1=T_l$ and $t_2\leqq\min(t_0, t_1+4.6\,T_l)$. In this model, the total ejected mass is smaller than $0.99\,M_i$. The condition of $(M_V/M)_{t=t_2}\geq 2$ reduces to

$$t_1\leq t_T{}^1\leq t_2-0.375\,T_l\,. \quad (34)$$

Then, from equations (30) and (32) we obtain

$$2\left(1+\frac{t_0-t_2}{1.375\,T_l}\right)\leq\frac{M_V}{M}\leq\frac{t_0}{T_l}\exp\left(\frac{t_2-t_1}{T_l}\right) \quad (35)$$

and

$$t_0-t_2+1.375\,T_l\leq T_{c,0}\leq t_0\,. \quad (36)$$

As in Case A, the relations of equations (21) and (22) hold also in this case. Then, from equation (36) the allowed range of H_{eff} is determined if $T_l(=t_1)$ and t_2 are known. In order to see the dependence of M_V/M upon t_2, the relation between M_V/M and $\bar{\rho}_0$ is illustrated in figure 3a for the case of $T_l=2.4\times10^9$ yr. Due to the condition (36), the region of $M_V/M\lesssim 10$ in the low-density side is forbidden.

In figure 3b, the time variation of M_V/M for various values of t_2 and $\alpha=T_l/T_{c,1}$ is illustrated for the case of $T_l=2.4\times10^9$ yr. As naturally expected, the growth of M_V/M becomes smaller with decreasing t_2. In figure 4, the relation between M_V/M and $\bar{\rho}_0$ is illustrated for various values of T_l. If T_l is fixed, the value of M_V/M in Case B is smaller than that in Case A for the same $\bar{\rho}_0$, because the duration time of mass-loss phase is short. However, it is possible to consider the case of T_l as small as, for example, 10^8 yr. In this case, the mass loss begins at $t_1=T_l=10^8$ yr after the formation of a cluster. If $\alpha=T_l/T_{c,1}=1$ and the duration of mass loss is $(t_2-t_1)>0.375t_l$, M_V/M increases even at $t>t_2$ and at the present time it reaches 1.6×10^4 for $(t_2-t_1)=4.6\,t_1$, and 182 for $(t_2-t_1)=0.375\,t_1$. This fact means that the mass ejection from a cluster at an early phase and at a rapid rate leads to a large virial mass discrepancy in the present time.

In the following Case C, we consider a case of mass loss at a rapid rate in an intermediate phase of evolution i.e., $t_1=10\,T_l$ and $t_2\leq\min(t_0, t_1+4.6\,T_l)$. Also in this case the relations (21) and (22) hold. From equations (33) and (31), the allowed ranges of $T_{c,0}$ and M_V/M become

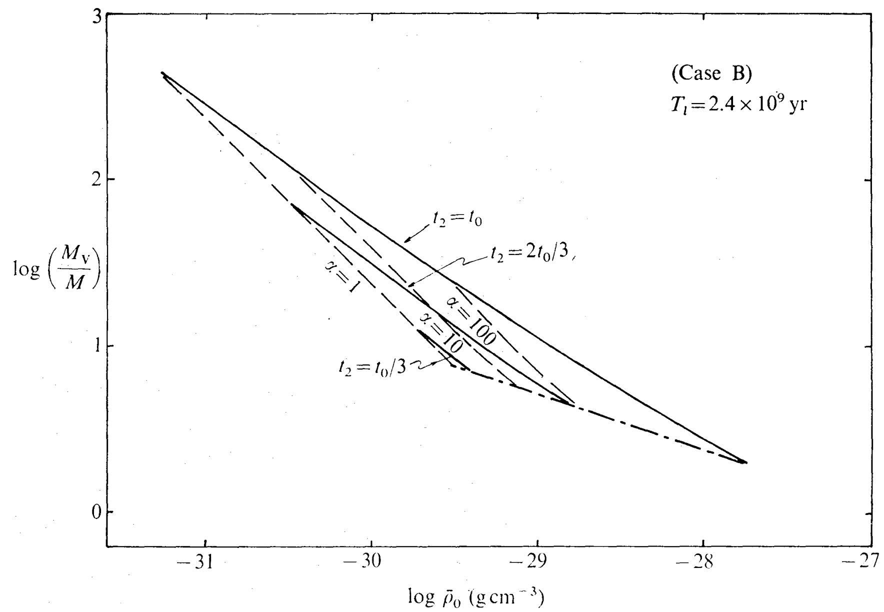

Fig. 3a. The same as in figure 2 but for Case B and $T_l=2.4\times10^9$ yr. The solid lines show the contours of constant t_2 and the dashed lines show those of constant α. The dot-dashed line shows the minimum values of M_V/M described in equation (35).

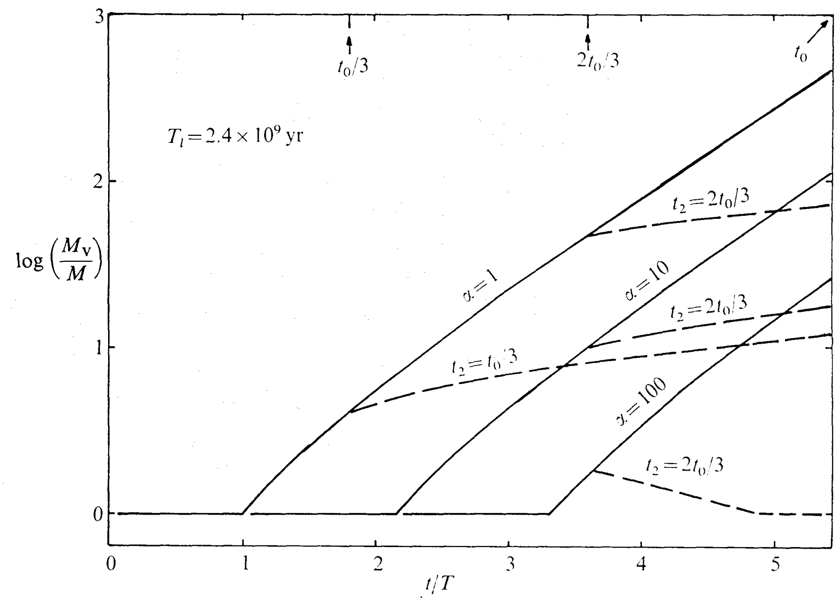

Fig. 3b. The time variation of the virial mass discrepancy for Case B and $T_l=2.4\times10^9$ yr. The solid lines show the case of $t_2=t_0$, and the dashed lines show the cases of $t_2=2t_0/3$ and $t_0/3$. In the case of $\alpha=100$ and $t_2=2t_0/3$, the value of M_V/M at $t=t_2$ is smaller than 2 so that it decreases.

$$t_0-9\,T_l\leq T_{c,0}\leq t_0 \tag{37}$$

and

$$\frac{t_0}{10\,T_l}\exp\left(\frac{t_2}{T_l}-10\right)\leq\frac{M_V}{M}\leq\left(\frac{t_0-9\,T_l}{T_l}\right)\exp\left(\frac{t_2}{T_l}-10\right), \tag{38}$$

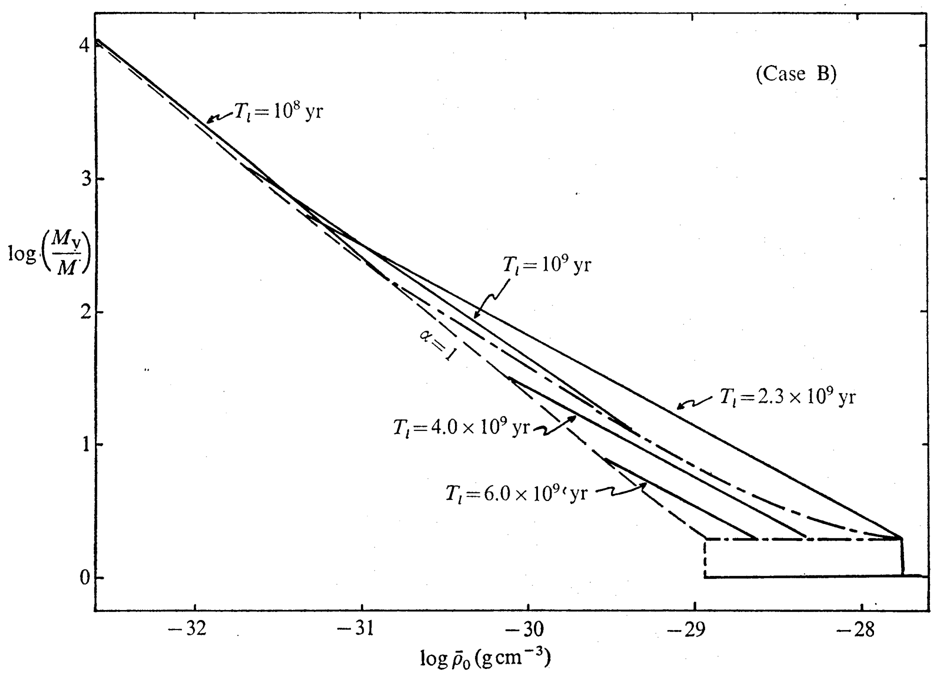

Fig. 4. The relation between the virial mass discrepancy and the density of the system for various values of T_l. In this figure, $t_2=\min(t_0, 5.6T_l)$ is assumed. The solid lines represent constant T_l. The dashed line shows the contour of $\alpha=1$, and the dot-dashed lines show the minimum values of M_V/M described in relation (35).

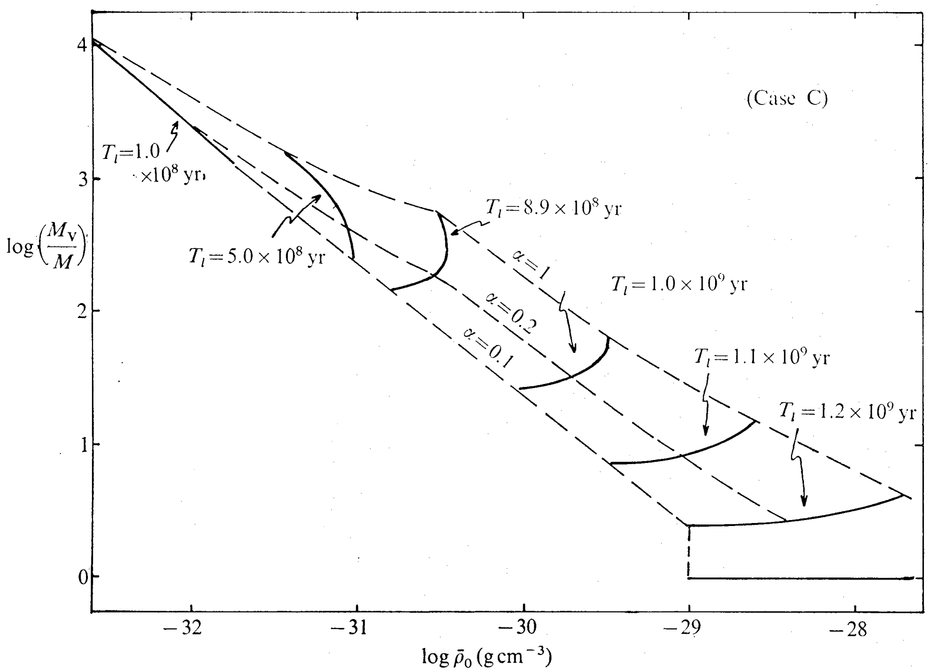

Fig. 5. The same as in figure 4 but for Case C. The solid lines show constant T_l, and the dashed lines show contours of constant α. In this figure, $t_2=\min(t_0, 14.6T_l)$ is assumed.

respectively. Since $(M_V/M)_{t=t_2}$ must be larger than two, the condition of

$$\left(1+\frac{t_2-t_1}{T_{c,i}}\right)\exp\left(\frac{t_2-t_1}{T_l}\right)>2 \tag{39}$$

must be satisfied. The dependence of M_V/M upon $\bar{\rho}_0$ at a constant T_l is different from two cases discussed in the above. In Case C, M_V/M increases with increasing α as in equation (31), but $T_{c,0}$ decreases with α in equation (33). Thus, $\bar{\rho}_0[\propto T_{c,0}^{-2}(M_V/M)^{-1}]$ increases with α at larger T_l but decreases at smaller T_l.

4. *Discussion*

As is shown above, the relation between the virial mass discrepancy and the density of a system presented by Hartwick (1978) is naturally explained in terms of the mass-loss hypothesis. Namely, systems with density lower than $10^{-28}\,\mathrm{g\,cm^{-3}}$ expand freely now. Their expansion rate is expressed by $H_{\mathrm{eff}}\simeq T_{c,0}^{-1}$, $T_{c,0}$ being described by parameters such as T_l, $\alpha(=T_l/T_{c,i})$, t_1, and t_2. If we suppose a mass-loss mechanism for a cluster, the range of allowed H_{eff} can be determined. On the other hand, systems with density higher than $10^{-28}\,\mathrm{g\,cm^{-3}}$ do not undergo the free expansion phase because of $T_{c,0}\lesssim T_l$.

The cluster model and the expansion law considered in this paper are too simple. Systems of galaxies are not uniform, but usually show centrally condensed distributions. Also, the transition stage from an adiabatic expansion to a free expansion is important as was shown by Aarseth and Saslaw (1972). Thus, the results obtained in this paper give upper limits of the virial mass discrepancy. However, the relation between M_V/M and $\bar{\rho}_0$ is valid and supported by observations. Then, it will be worthwhile to study the relation between the richness of clusters and the virial mass discrepancy more carefully in future.

It is to be noted that in the above calculations the values of mass and radius of the system are not directly used. Essentially, the virial mass discrepancy is determined by the age of the system, the crossing time at the initial time, and the mass-loss parameters. Thus, the above considerations are also applicable to a stellar cluster or a galaxy. In an open cluster or a stellar association, $T_{c,i}$ will be fairly long because of its diffuse structure. As massive stars in a stellar cluster evolve faster, the remaining gas is ejected out rapidly ($T_l<T_{c,i}$) by their radiation pressure. [As Noerdlinger (1970) has proposed, QSOs in a group of galaxies seem to play a similar role.] In a globular cluster, $T_{c,i}$ will be shorter than T_l because of its compactness.

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